## One Plus One Equals Two

0) $\forall a: \forall b:(a+S b)=S(a+b)$ [axiom]
1) $\forall b:(S 0+S b)=S(S 0+b) \quad$ [specification of a to $\mathbf{S 0}$ in theorem 0 ]
2) $(S 0+S 0)=S(S 0+0) \quad$ [specification of $\mathbf{b}$ to $\mathbf{0}$ in theorem 1]
3) $\forall a:(a+0)=a \quad$ [axiom]
4) $(S 0+0)=S 0 \quad$ [specification of a to $\mathbf{S 0}$ in theorem 3]
5) $S(S 0+0)=S S 0 \quad$ [successor of theorem 4]
6) $(S 0+S 0)=S S 0 \quad$ [transitivity of theorem 2 and theorem 5]
