

# Transparent Inner Product

Bulletproofs[1] create arguments of knowledge of the inner product of two vectors  $\vec{a}, \vec{b} \in \mathbb{Z}_q^m$  with prime  $q$ , denoted  $\langle \vec{a}, \vec{b} \rangle \in \mathbb{Z}_q$ . This is hadamard multiplication, followed by sum, yielding a scalar value. Bulletproofs have  $\mathcal{O}(\log(m))$  argument size, and  $\mathcal{O}(m)$  proving/verification complexity.

However, to achieve hiding, bulletproofs require lightweight (scalar) linear homomorphism. This approach works poorly with post quantum arguments where hidden linear operations are much more expensive. However, a binding only variant is quite cheap using scalar SIS commitments to public coin algebraic structure.

This work proposes using the bulletproof algebraic inner product argument to achieve succinct, *binding only*, arguments of knowledge. This is designed to be composed on top of a module lattice argument of knowledge, which provides *hiding and binding*. Because the inner product argument is strictly logarithmic, scaling to statistical hiding in the module lattice construction has little practical impact on the final argument size.

## 0.1 TODO

Scalar SIS derivation

## 1 Argument

Given  $\vec{a}, \vec{b} \in \mathbb{Z}_q^m$  yield  $c \in \mathbb{Z}_q, \pi \in \mathbb{Z}_q^{\log(m)}$ , arguing  $\langle \vec{a}, \vec{b} \rangle = c$ . Denote  $\mathcal{H} : \mathbb{Z}_q^{\mathbb{Z}^+} \rightarrow \mathbb{Z}_q$  as a collision and pre-image resistant hash function. Prover computes:

$$c \leftarrow \langle \vec{a}, \vec{b} \rangle$$
$$transcript : \vec{t} \stackrel{\$}{\leftarrow} \vec{a}, \vec{b}$$

$$\pi_k \leftarrow \sum_{i=0}^{\log(m)}$$

## References

- [1] Benedikt Bünz and Jonathan Bootle and Dan Boneh and Andrew Poelstra and Pieter Wuille and Greg Maxwell, Bulletproofs: Short Proofs for Confidential Transactions and More, Cryptology ePrint Archive, Paper 2017/1066, 2017, <https://eprint.iacr.org/2017/1066>