Validation Architecture in delaunay

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Author TODO. Write the abstract in your own words.

Additional Key Words and Phrases: TODO author-supplied keywords

1 INTRODUCTION

- **Author TODO.** State the problem and motivation.
- **Author TODO.** Explain why validation architecture matters for a scientific computational-geometry library.
- **Author TODO.** Summarize the central contribution in your own words.
- **Author TODO.** Preview the paper structure.

Applications such as Causal Dynamical Triangulation (CDT) require a robust and reliable triangulation library, capable of maintaining correct properties of the triangulation in simulations that may alter their structure millions of times per simulation. These invariant properties should be verifiable during construction and mutation, and violations of these properties should be detectable and diagnosable, so that fixable errors may be corrected and unfixable errors may be reported.

Delaunay is a Rust library for constructing and mutating triangulations with insertion, deletion, and bi-stellar k-flip operations, while maintaining a range of invariant properties important for the construction of Pseudo- and Piecewise-Linear (PL) manifolds, the evaluation of geometric predicates, and the embedding of triangulations in Euclidean, toroidal, and spherical spaces.

Author TODO. Qualify the preceding overview so the ordinary mutable Euclidean/toroidal workflows are distinguished from the separate bounded S^2/S^3 spherical prototype.

2 BACKGROUND AND DEFINITIONS

- **Author TODO.** Define the triangulation, simplex, ridge, facet, and realization terms used throughout the paper.
- **Author TODO.** Introduce PL-manifold terminology and the link condition.
- **Author TODO.** Introduce Euclidean, toroidal, and spherical realization models.
- **Author TODO.** Introduce Delaunay and future geometric-predicate terminology.

3 VALIDATION HIERARCHY

3.1 Level 1: Element Validity

- **Author TODO.** Describe what individual vertices, simplices, facets, and coordinates must prove locally.
- **Author TODO.** Explain what this level deliberately does not check.

3.2 Level 2: Combinatorial Consistency

- **Author TODO.** Describe adjacency, incidence, neighbor symmetry, and TDS integrity.
- **Author TODO.** Distinguish coherent stored simplex orderings, including periodic quotient facets, from intrinsic orientability.
- **Author TODO.** Explain the separation from topology and geometry.

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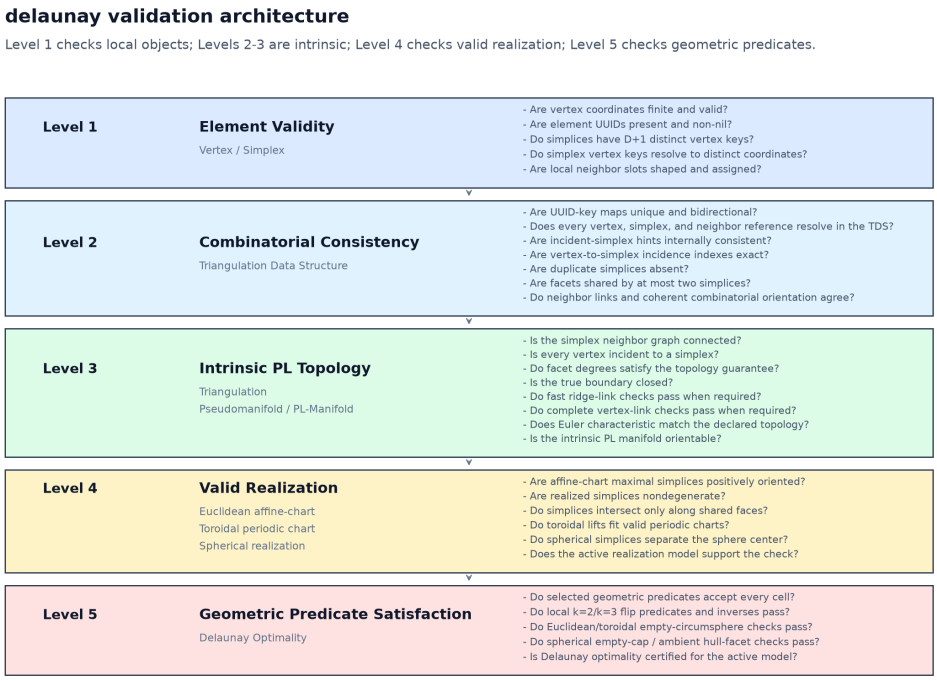

Fig. 1. Overview of the validation hierarchy in delaunay

3.3 Level 3: Intrinsic PL Topology

- **Author TODO.** State the PL-manifold and boundary conditions.
- **Author TODO.** Discuss pseudomanifold and strict PL-manifold policy choices.
- **Author TODO.** Describe the 2D/3D intrinsic orientation witness and periodic quotient parity constraints.
- **Author TODO.** Explain why this layer is independent of Euclidean, toroidal, and spherical realizations.

3.4 Level 4: Valid Realization

- **Author TODO.** Use the terminology stack: Level 4 Valid Realization; Euclidean/toroidal valid affine-chart realization; spherical valid spherical realization; Level 5 Geometric Predicate Satisfaction / Delaunay Optimality.
- **Author TODO.** Describe valid realization in the active ambient model.
- **Author TODO.** Compare Euclidean/toroidal affine-chart realizations and spherical realizations.
- **Author TODO.** Keep positive geometric orientation distinct from Level 2 stored coherence and Level 3 intrinsic orientability.
- **Author TODO.** Explain why realization validity is a precondition for geometric predicates.

- **Author TODO.** Introduce two realized d-simplices with vertices $a_0, \ldots, a_d$ and $b_0, \ldots, b_d$, vertex labels ℓ_i and m_j, and shared-label set S, then explain the exact shared-face intersection program in Equation 1.

$$\begin{aligned} z^\star = \underset{\alpha_i, \beta_j \geq 0}{\text{maximize}} \quad & \sum_{\ell_i \notin S} \alpha_i + \sum_{m_j \notin S} \beta_j \\ \text{subject to} \quad & \sum_{i=0}^{d} \alpha_i a_i = \sum_{j=0}^{d} \beta_j b_j, \\ & \sum_{i=0}^{d} \alpha_i = 1, \qquad \sum_{j=0}^{d} \beta_j = 1. \end{aligned} \tag{1}$$

- **Author TODO.** Explain the three exact outcomes: infeasibility proves the simplices disjoint; $z^\star = 0$ proves every feasible intersection point is confined to the shared face; and $z^\star > 0$ supplies barycentric weight on non-shared labels for a typed intersection witness.
- **Author TODO.** Explain how revised-simplex basis navigation replaces exhaustive active-set enumeration of the intersection polytope on the common path and removes the bottleneck exposed by the 4D and 5D Level 4 checks.
- **Author TODO.** Describe the filtered-exact hierarchy: bounding-box, separating-facet, orientation, and shared-face normal tests reject or certify easy pairs; floating-point simplex phases propose axes and bases; proved floating-point error bounds or exact rational primal/dual checks certify accepted results; and active-set enumeration remains the conservative fallback.
- **Author TODO.** Describe how exact finite-`f64` systems use the runtime-dispatched fraction-free Bareiss solver from `la-stack`, while general rational systems retain the `BigRational` fallback.
- **Author TODO.** Treat the broader independent randomized and degenerate 2D–5D agreement campaign and representative before/after benchmarks as v0.8.1 work tracked by issue #482.
- **Author TODO.** Do not present the bounded v0.8.0 4D and 5D slow-test probes as performance evidence. Coordinate any Level 5 all-violations performance claims with the separate v0.8.1 work in issue #483.

3.5 Level 5: Geometric Predicates

- **Author TODO.** Describe Delaunay as the first implemented predicate family.
- **Author TODO.** Explain how Euclidean, toroidal, and spherical Delaunay predicates differ.
- **Author TODO.** Leave room for regular, weighted, constrained, Gabriel, alpha, and future predicate families.

4 IMPLEMENTATION ARCHITECTURE

Table 1. Public validation API by owning layer.

Level	Canonical public surface
1	`Vertex::is_valid` / `vertex_report`; `Simplex::is_valid` / `simplex_report`
2	`Tds::is_valid` / `structure_report`; `Tds::validate` certifies Levels 1–2
3	`Triangulation::is_valid_topology` / `topology_report` / `orientation_witness`; `Triangulation::validate` certifies Levels 1–3
4	`Triangulation::is_valid_realization` / `realization_report`; `validate_realization` certifies Levels 1–4
5	`DelaunayTriangulation::is_valid_delaunay` / `delaunay_report`; `DelaunayTriangulation::validate` certifies Levels 1–5

- **Author TODO.** Map the five validation levels onto the Rust API surface.
- **Author TODO.** Describe the role of fast-fail checks, diagnostics, reports, and cumulative validation.
- **Author TODO.** Explain construction-time validation policy choices such as `ExplicitOnly` and `OnSuspicion`.
- **Author TODO.** Describe how spherical topology fits as a coordinate/metric backend rather than a new simplex dimension.

5 DIAGNOSTICS AND REPRODUCIBLE FIGURES

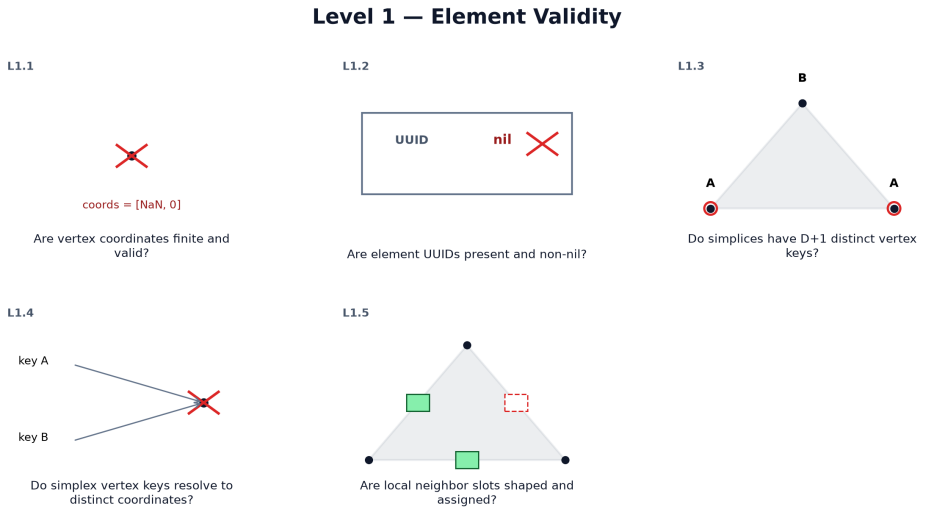

Fig. 2. **Author TODO.** Describe the Level 1 validation witness figure.

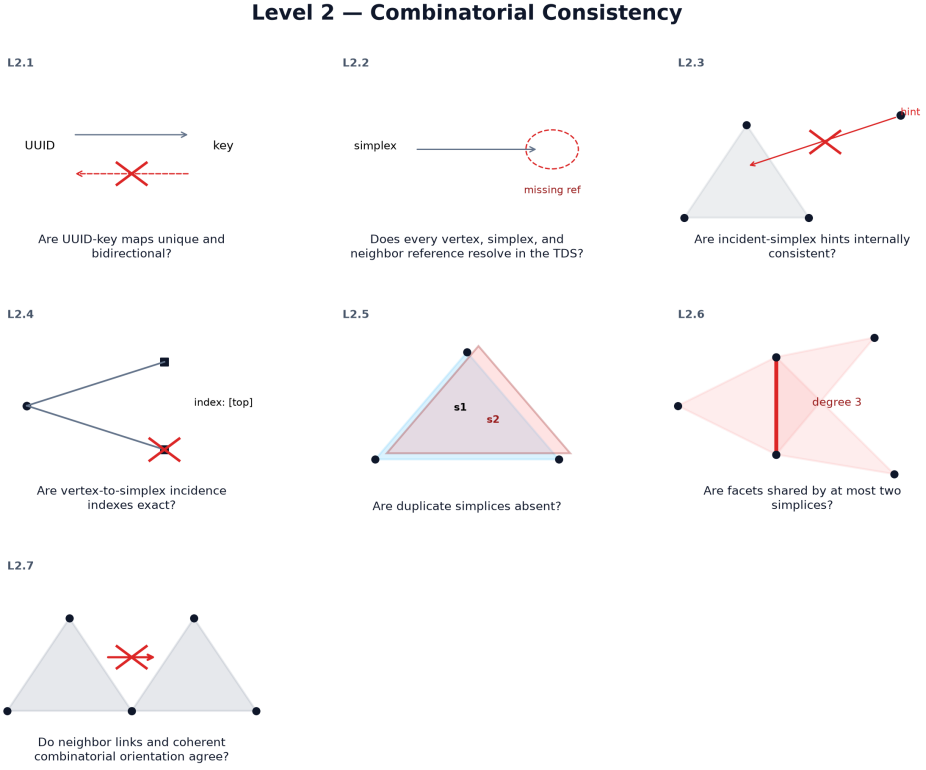

Fig. 3. **Author TODO.** Describe the Level 2 validation witness figure.

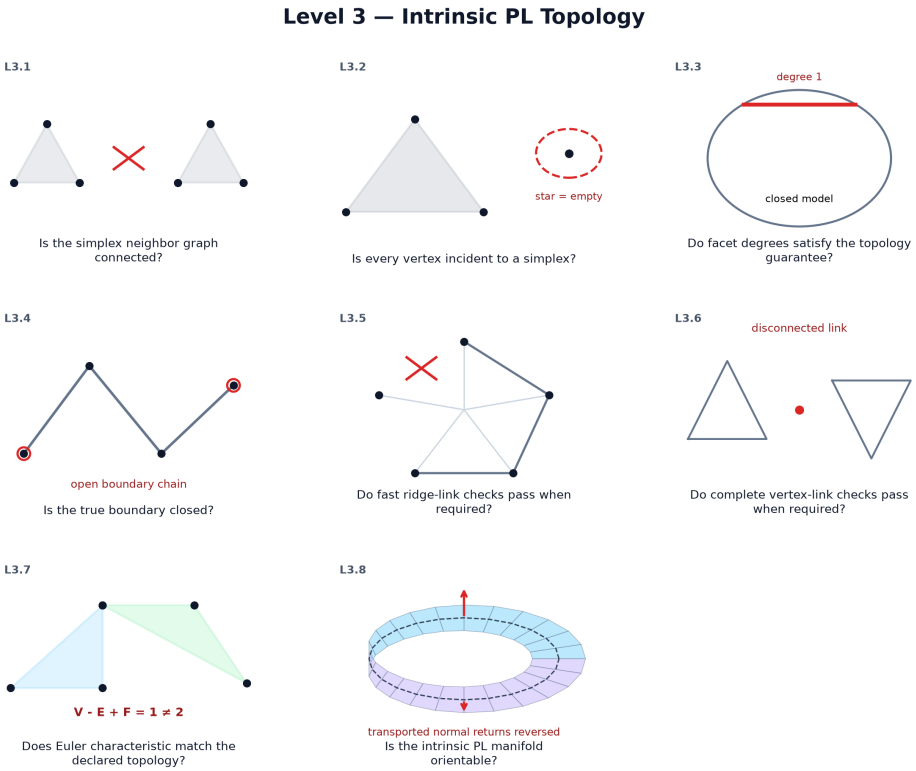

Fig. 4. **Author TODO.** Describe the Level 3 validation witness figure.

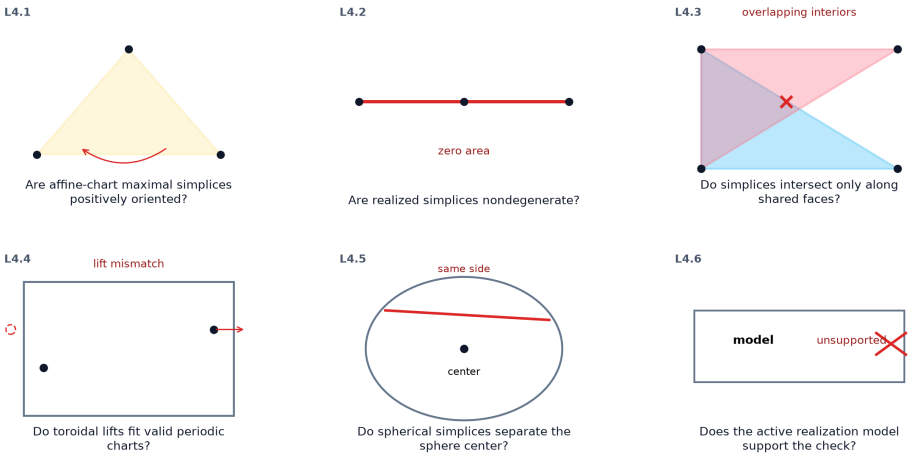

Fig. 5. **Author TODO.** Describe the Level 4 validation witness figure.

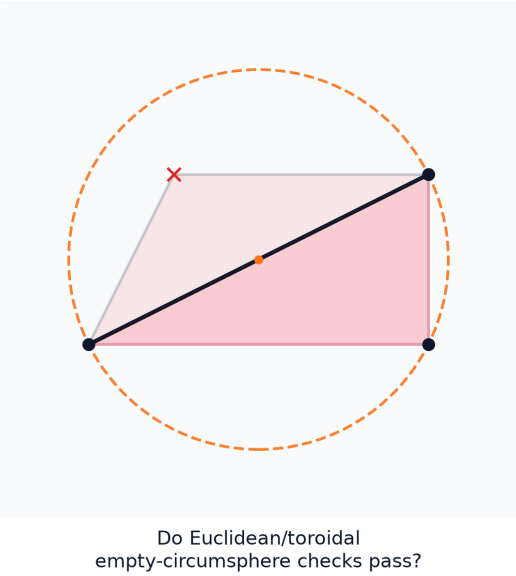

Fig. 6. **Author TODO.** Describe the Level 5 validation witness figure.

- **Author TODO.** Explain how the validation notebook generates paper figures.
- **Author TODO.** Describe representative diagnostics without duplicating API documentation.
- **Author TODO.** Connect figure generation to the reproducibility story.

6 RELATED WORK

- **Author TODO.** Place PL topology references and Pachner/Lickorish results.
- **Author TODO.** Place Euclidean robust-predicate and Delaunay references.
- **Author TODO.** Place spherical Delaunay and Riemannian-manifold references.
- **Author TODO.** Clarify which claims are architectural observations rather than literature claims.

7 DISCUSSION AND FUTURE WORK

- **Author TODO.** Discuss extensibility to additional realization models and predicate families.
- **Author TODO.** Discuss limitations of the current implementation.
- **Author TODO.** Identify open validation and benchmarking questions.

8 CONCLUSION

- **Author TODO.** Write the conclusion in your own words.

ACKNOWLEDGMENTS

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REFERENCES

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